EXAMPLE 1. *Find the range and the coefficient of range for the following observations:*

65, 70, 82, 59, 81, 76, 57, 60, 55 and 50 **[CA PEE-I, Nov. 2003]**

SOLUTION. Range = $L - S$

Here, $L = 82$ and $S = 50$

$\therefore$ Range $= 82 - 50 = 32$

$$\text{Coefficient of range} = \frac{L-S}{L+S} = \frac{82-50}{82+50} = \frac{32}{132} = 0.24.$$

EXAMPLE 2. *Calculate range and coefficient of range from the following data:*

Marks :	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of Students :	2	6	12	18	25	20	10	7

SOLUTION. Range $= L - S = 90 - 10 = 80$

$$\text{and Coefficient of range} = \frac{L-S}{L+S} = \frac{90-10}{90+10} = \frac{80}{100} = 0.8.$$

EXAMPLE 3. *Find the quartile deviation of the daily wages (in Rs.) of 7 persons given below:*

120	*70*	*150*	*100*	*190*	*170*	*250*

SOLUTION. Arranging the data in ascending order of magnitude, we get

70	100	120	150	170	190	250

Here, $n = 7,\ \dfrac{n+1}{4} = \dfrac{7+1}{4} = 2,\ \dfrac{3(n+1)}{4} = 6$

$$\therefore \qquad Q_1 = \text{size of } \left(\frac{n+1}{4}\right)\text{th item} = \text{size of second item} = 100$$

$$Q_3 = \text{size of } \frac{3(n+1)}{4}\text{th item} = \text{size of sixth item} = 190$$

Hence Quartile Deviation $= \dfrac{Q_3 - Q_1}{2} = \dfrac{190-100}{2} = 45.$

EXAMPLE 4. *Calculate the quartile deviation for the following frequency distribution :*

X :	*60*	*62*	*64*	*66*	*68*	*70*	*72*
f :	*12*	*16*	*18*	*20*	*15*	*13*	*9*

SOLUTION.

CALCULATION FOR QUARTILES

X	*f*	*c.f.*
60	12	12
62	16	28
64	18	46
66	20	66
68	15	81
70	13	94
72	9	103
	$N = \sum f = 103$	

Here, $N = 103, \quad \dfrac{N+1}{4} = \dfrac{104}{4} = 26$ and $\dfrac{3(N+1)}{4} = 78$

$\therefore \quad Q_1 = \text{size of } \dfrac{N+1}{4}\text{th item} = \text{size of 26th item} = 62$

$Q_3 = \text{size of } \dfrac{3(N+1)}{4}\text{th item} = \text{size of 78th item} = 68$

Hence Quartile Deviation $= \dfrac{Q_3 - Q_1}{2} = \dfrac{68-62}{2} = 3.$

EXAMPLE 5. *Calculate quartile deviation from the following distribution :*

X :	*5 – 7*	*8 – 10*	*11 – 13*	*14 – 16*	*17 – 19*
Frequency :	*14*	*24*	*38*	*20*	*4*

SOLUTION.

CALCULATIONS FOR QUARTILES

Class Boundary	*Frequency (f)*	*Less than c.f.*
4.5 - 7.5	14	14
7.5 - 10.5	24	38
10.5 - 13.5	38	76
13.5 - 16.5	20	96
16.5 - 19.5	4	100
	$N = \sum f = 100$	

Here it is requires distribution to be continuous with exclusive type'

***Computation of Q_1*:** We have $\frac{N}{4} = \frac{100}{4} = 25$; and the c.f. just greater than or equal to 25 is 38. The class corresponding to this c.f. is 7.5 – 10.5. Thus Q_1 – class is 7.5 – 10.5.

$$\therefore \quad Q_1 = l + \frac{\frac{N}{4} - c.f}{f} \times h = 7.5 + \frac{25 - 14}{24} \times 3 = 7.5 + 1.375 = 8.875$$

***Computation of Q_3*:** $\frac{3N}{4} = 75$; and the c.f. just greater than or equal to 75 is 76. The class corresponding to this c.f. is 10.5 – 13.5. Thus Q_3 – class is 10.5 – 13.5.

$$\therefore \quad Q_3 = l + \frac{\frac{3N}{4} - c.f}{f} \times h = 10.5 + \frac{75 - 38}{38} \times 3 = 10.5 + 2.92 = 13.42$$

Computation of Quartile Deviation:

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{13.42 - 8.875}{2} = 2.27 \text{ (app.)}$$

EXAMPLE 8. *Find the inter-quartile range and the coefficient of quartile deviation from the following data:*

Marks less than :	*10*	*20*	*30*	*40*	*50*	*60*	*70*	*80*
No. of Students :	*4*	*16*	*40*	*76*	*96*	*112*	*120*	*125*

SOLUTION. We shall first convert the given cumulative frequency distribution into an ordinary frequency distribution.

Marks	*f*	*c.f.*
0 – 10	4	4
10 – 20	12	16
20 – 30	24	40
30 – 40	36	76
40 – 50	20	96
50 – 60	16	112
60 – 70	8	120
70 – 80	5	125
	$N = 125$	

Computation of Q_1 : $\frac{N}{4} = \frac{125}{4} = 31.25$; the c.f. just greater than or equal to 31.25 is 40.

Therefore Q_1 lies in the class 20 – 30 and is given by

$$Q_1 = l + \frac{\frac{N}{4} - C}{f} \times h = 20 + \frac{31.25 - 16}{24} \times 10 = 26.35.$$

Computation of Q_3: $\frac{3N}{4} = 93.75$; the *c.f.* just greater than or equal to 93.75 is 96. Therefore Q_3 lies in the class 40 – 50 and is given by

$$Q_3 = l + \frac{\frac{3N}{4} - C}{f} \times h = 40 + \frac{93.75 - 76}{20} \times 10 = 48.88$$

$\therefore$ Inter-quartile range $= Q_3 - Q_1 = 48.88 - 26.35 = 22.53$

and Coefficient of Q.D. $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{48.88 - 26.35}{48.88 + 26.35} = \frac{22.53}{75.23} = 0.30$ (app.)

Find also the coefficient of mean ...

SOLUTION. Arranging the data in ascending order of magnitude, we get

26 29 39 46 56 65 72 79 85 9...

Here, $n = 10$

$$\therefore \quad \frac{n+1}{2} = \frac{10+1}{2} = 5.5$$

$$\therefore \quad \text{Median} = \text{size of} \left(\frac{n+1}{2}\right)\text{th item}$$

$$= \text{size of (5.5)th item}$$

$$= \text{5th item} + 0.5 \text{ (6th item} - \text{5th item)}$$

$$= 56 + 0.5\,(65 - 56) = 56 + 4.5 = 60.5$$

56 + 65 / 2 = 121 / 2 = 60.5

CALCULATION OF MEAN DEVIATION

X	Deviation from Median $D = X - Md$	Absolute Deviation from Median $\|D\|$
26	-34.5	34.5
29	-31.5	31.5
39	-21.5	21.5
46	-14.5	14.5
56	-4.5	4.5
65	4.5	4.5
72	11.5	11.5
79	18.5	18.5
85	24.5	24.5
99	38.5	38.5
$n = 10$		$\sum\|D\| = 204$

$$\therefore \quad M.D. \text{ (about median)} = \frac{\sum|D|}{n} = \frac{204}{10} = 20.4.$$

$$\text{Coefficient of } M.D. = \frac{\text{Mean Deviation}}{\text{Median}} = \frac{20.4}{60.5} = 0.34.$$

EXAMPLE 10. *Calculate mean deviation about the mean for the following data:*

X	:	*10*	*11*	*12*	*13*	*14*	*Total*
f	:	*3*	*12*	*18*	*12*	*3*	*48*

SOLUTION. **CALCULATIONS FOR MEAN DEVIATION**

X	f	fX	$D = X - \bar{X}$	$\|D\|$	$f\|D\|$
10	3	30	−2	2	6
11	12	132	−1	1	12
12	18	216	0	0	0
13	12	156	1	1	12
14	3	42	2	2	6
	$N = \sum f = 48$	$\sum fX = 576$			$\sum f\|D\| = 36$

$$\text{Mean} : \bar{X} = \frac{\sum fX}{\sum f} = \frac{576}{48} = 12$$

$$\text{Mean Deviation about Mean} = \frac{\sum f|D|}{N} = \frac{36}{48} = 0.75.$$

EXAMPLE 11. *Calculate the mean deviation from the median for the following data:*

Marks	:	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*	*60 – 70*	*70 – 80*	*80 – 90*
No. of Students	:	*2*	*6*	*12*	*18*	*25*	*20*	*10*	*7*

Also calculate the coefficient of mean deviation from median. **[Delhi Univ. B.Com.**

SOLUTION. **CALCULATION OF MEAN DEVIATION**

Marks	Mid-value X	No. of Students f	Less than cf	D	$\lvert D \rvert$	$f\lvert D \rvert$
10 – 20	15	2	2	–39.8	39.8	79.6
20 – 30	25	6	8	–29.8	29.8	178.8
30 – 40	35	12	20	–19.8	19.8	237.6
40 – 50	45	18	38	–9.8	9.8	176.4
50 – 60	55	25	63	0.2	0.2	5.0
60 – 70	65	20	83	10.2	10.2	204.0
70 – 80	75	10	93	20.2	20.2	202.0
80 – 90	85	7	100	30.2	30.2	211.4
		$N = \sum f = 100$				$\sum f\lvert D \rvert = 1294.8$

Computation of Median : We have $\frac{N}{2} = 50$. The c.f. just greater than or equal to 50 is 63 and the corresponding class interval is 50 – 60. Thus the median class is 50 – 60.

$$\therefore \quad \text{Median } (Md) = l + \frac{\frac{N}{2} - C}{f} \times h = 50 + \frac{50 - 38}{25} \times 10$$

$$= 50 + \frac{120}{25} = 50 + 4.8 = 54.8.$$

Computation of Mean Deviation :

$$M.D. = \frac{\sum f \lvert D \rvert}{N} = \frac{1294.8}{100} = 12.948.$$

$$\therefore \quad \text{Coefficient of } M.D. \text{ from median} = \frac{M.D. \text{ (about median)}}{\text{Median}} = \frac{12.948}{54.8} = 0.24$$

EXAMPLE 12. *Calculate the mean deviation from the mean for the following data:*

Marks :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students :	6	5	8	15	7	6

[C.A. Foundation, May

SOLUTION. **CALCULATION OF MEAN DEVIATION FROM MEAN**

Marks	Mid-value X	No. of Students f	fX	$D = X - \bar{X}$	|D|	f|D|
0 – 10	5	6	30	–28.4	28.4	170.4
10 – 20	15	5	75	–18.4	18.4	92.0
20 – 30	25	8	200	–8.4	8.4	67.2
30 – 40	35	15	525	1.6	1.6	24.0
40 – 50	45	7	315	11.6	11.6	81.2
50 – 60	55	6	330	21.6	21.6	129.6
60 – 70	65	3	195	31.6	31.6	94.8
		$N = 50$	$\sum fX = 1670$			$\sum f\|D\| = 659.2$

Computation of Mean:

$$\bar{X} = \frac{\sum fX}{N} = \frac{1670}{50} = 33.4$$

Computation of Mean Deviation about Mean:

$$M.D. = \frac{\sum f|D|}{N} = \frac{659.2}{50} = 13.18 \text{ (app.).}$$

EXAMPLE 13. *Calculate the mean deviation and its coefficient.*

Marks :	*21 – 25*	*26 – 30*	*31 – 35*	*36 – 40*	*41 – 45*	*46 – 50*	*51 – 55*
No. of Students :	*5*	*15*	*28*	*42*	*15*	*12*	*3*

[Delhi Univ. B.Com. 2004]

SOLUTION. The mean deviation gives the best results when deviations are taken from median. Since nothing is specified in the question, we shall take deviations from median.

CALCULATION OF MEAN DEVIATION

Marks	Mid-Value X	No. of Students f	c.f.	$D = X - Md$	|D|	f|D|
20.5 – 25.5	23	5	5	–13.93	13.93	69.65
25.5 – 30.5	28	15	20	– 8.93	8.93	133.95
30.5 – 35.5	33	28	48	–3.93	3.93	110.04
35.5 – 40.5	38	42	90	1.07	1.07	44.94
40.5 – 45.5	43	15	105	6.07	6.07	91.05
45.5 – 50.5	48	12	117	11.07	11.07	132.84
50.5 – 55.5	53	3	120	16.07	16.07	48.21
		$N = \sum f = 120$				$\sum f\|D\| = 630.68$

Computation of Median :

$$\frac{N}{2} = \frac{120}{2} = 60$$; the *c.f.* just greater than or equal to 60 is 90.

Thus median lies in the class 35.5 - 40.5 and is given by

$$\text{Median } (Md) = l + \frac{\frac{N}{2} - C}{f} \times h = 35.5 + \frac{60 - 48}{42} \times 5 = 35.5 + 1.43 = 36.93$$

$$\therefore \text{ M.D. (about Median)} = \frac{\sum f \mid D \mid}{N} = \frac{630.68}{120} = 5.26$$

$$\text{Coefficient of Mean Deviation (about median)} = \frac{M.D.}{\text{Median}} = 0.142.$$

EXAMPLE 14. *Calculate standard deviation of the following marks obtained by 5 students in a tutorial group:*

Marks obtained : 8 12 13 15 22 **[*Delhi Univ. B.Com. 1997*]**

SOLUTION. **CALCULATION OF STANDARD DEVIATION**

Marks obtained (X)	$X-\bar{X}$	$(X-\bar{X})^2$
8	−6	36
12	−2	4
13	−1	1
15	1	1
22	8	64
$\sum X = 70$		$\sum(X-\bar{X})^2 = 106$

$$\therefore \quad \bar{X} = \frac{\sum X}{n} = \frac{70}{5} = 14$$

and
$$\sigma = \sqrt{\frac{\sum(X-\bar{X})^2}{n}} = \sqrt{\frac{106}{5}} = \sqrt{21.2} = 4.6 \text{ (app.)}.$$

Thus the standard deviation of wages for the group of 10 workers is Rs. 6.

EXAMPLE 16. *Calculate the standard deviation for the following data:*

X	:	20	30	40	50	60	70
f	:	8	12	20	10	6	4

SOLUTION. **CALCULATION OF STANDARD DEVIATION**

X	f	fX	$X-\bar{X}$	$(X-\bar{X})^2$	$f(X-\bar{X})^2$
20	8	160	-21	441	3528
30	12	360	-11	121	1452
40	20	800	-1	1	20
50	10	500	9	81	810
60	6	360	19	361	2166
70	4	280	29	841	3364
	$N=60$	$\sum fX=2460$			$\sum f(X-\bar{X})^2=11{,}340$

$$\therefore \quad \bar{X}=\frac{\sum fX}{N}=\frac{2460}{60}=41$$

and
$$\sigma=\sqrt{\frac{\sum f(X-\bar{X})^2}{N}}=\sqrt{\frac{11340}{60}}=\sqrt{189}=13.75.$$

EXAMPLE 17. *Calculate the mean and standard deviation from the following data:*

Marks :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
No. of Students :	10	15	25	25	10	10	5

SOLUTION. **CALCULATION OF STANDARD DEVIATION**

Marks	*Mid value* X	*No. of Students* f	fX	$X-\bar{X}$	$(X-\bar{X})^2$	$f(X-\bar{X})^2$
0 – 10	5	10	50	-26	676	6760
10 – 20	15	15	225	-16	256	3840
20 – 30	25	25	625	-6	36	900
30 – 40	35	25	875	4	16	400
40 – 50	45	10	450	14	196	1960
50 – 60	55	10	550	24	576	5760
60 – 70	65	5	325	34	1156	5780
		$N=100$	$\sum fX=3100$			$\sum f(X-\bar{X})^2=25{,}400$

$$\therefore \qquad \bar{X} = \frac{\sum fX}{N} = \frac{3100}{100} = 31$$

and

$$\sigma = \sqrt{\frac{\sum f(X-\bar{X})^2}{N}} = \sqrt{\frac{25400}{100}} = \sqrt{254} = 15.94.$$

$\sqrt{\ldots N \quad \left(\ldots N\right)} \ldots$ where $u = \frac{\ldots}{h}$

EXAMPLE 18. *From the following information, find the standard deviation of X and Y variable:*

$\sum X = 235, \quad \sum Y = 250, \quad \sum X^2 = 6750, \quad \sum Y^2 = 6840, \quad N = 10$

[Delhi Univ. B.Com. (H) 1997]

SOLUTION. ***Standard Deviation of X variable.***

$$\sigma_X = \sqrt{\frac{\sum X^2}{N} - \left(\frac{\sum X}{N}\right)^2} = \sqrt{\frac{6750}{10} - \left(\frac{235}{10}\right)^2}$$

$$= \sqrt{675 - (23.5)^2} = \sqrt{675 - 552.25} = \sqrt{122.75} = 11.08$$

Standard deviation of Y variable.

$$\sigma_Y = \sqrt{\frac{\sum Y^2}{N} - \left(\frac{\sum Y}{N}\right)^2} = \sqrt{\frac{6840}{10} - \left(\frac{250}{10}\right)^2} = \sqrt{684 - (25)^2}$$

$$= \sqrt{684 - 625} = \sqrt{59} = 7.68.$$

Imp

EXAMPLE 19. *Find the mean and standard deviation of first n natural numbers.*

SOLUTION. The first n natural numbers are 1, 2, 3, ..., n. We know that

$$\sum X = 1 + 2 + 3 + \ldots. + n = \frac{n(n+1)}{2}$$

and $$\sum X^2 = 1^2 + 2^2 + 3^2 + \ldots. + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$\therefore$ $$\text{Mean} = \bar{X} = \frac{\sum X}{n} = \frac{1+2+3+\ldots+n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

and $$S.D. = \sigma = \sqrt{\frac{\sum X^2}{n} - \bar{X}^2} = \sqrt{\frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2}$$

$$= \sqrt{\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}} = \sqrt{\frac{2(n+1)(2n+1) - 3(n+1)^2}{12}}$$

$$= \sqrt{\frac{(n+1)[4n+2-3n-3]}{12}} = \sqrt{\frac{(n+1)(n-1)}{12}} = \sqrt{\frac{n^2-1}{12}}$$

Thus, the mean of first n natural numbers is $\frac{n+1}{2}$ and their standard deviation is $\sqrt{\frac{n^2-1}{12}}$.

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{i=1}^{n} (i) = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

EXAMPLE 22. *Calculate the arithmetic mean and standard deviation from the following series.*

Class Interval	:	*5 – 15*	*15 – 25*	*25 – 35*	*35 – 45*	*45 – 55*
Frequency	:	*8*	*12*	*15*	*9*	*6*

[*Delhi Univ. B.Com. 1973*]

SOLUTION.

CALCULATION OF A.M. AND S.D.

Class Interval	*Mid-value X*	*Frequency f*	$u = \frac{X-A}{h}$ *(A = 30, h = 10)*	*fu*	fu^2
5 – 15	10	8	– 2	– 16	32
15 – 25	20	12	– 1	– 12	12
25 – 35	30	15	0	0	0
35 – 45	40	9	1	9	9
45 – 55	50	6	2	12	24
		$N = 50$		$\sum fu = 7$	$\sum fu^2 = 77$

Calculation of Arithmetic Mean. The *A.M.* is given by the formula

$$\bar{X} = A+\frac{\Sigma fu}{N}\times h = 30+\frac{-7}{50}\times 10 = 30-1.4 = 28.6.$$

Calculation of Standard Deviation. The S.D. is given by the formula

$$\sigma = \sqrt{\frac{\Sigma fu^2}{N}-\left(\frac{\Sigma fu}{N}\right)^2}\times h = \sqrt{\frac{77}{50}-\left(\frac{-7}{50}\right)^2}\times 10$$

$$= \sqrt{1.54-0.0196}\times 10 = 12.33.$$

EXAMPLE 28. *From the following information, calculate the combined standard deviation:*

$$n_1 = 90 \quad \bar{X}_1 = 20 \quad \sigma_1 = 8$$

$$n_2 = 60 \quad \bar{X}_2 = 15 \quad \sigma_2 = 6$$ **[Delhi Univ. B.Com. 2004]**

SOLUTION. Let $\bar{X}$ denote the combined mean and σ denote the combined standard deviation. Then

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2} = \frac{90\times20+60\times15}{90+60} = \frac{1800+900}{150} = \frac{2700}{150} = 18$$

and

$$\sigma = \sqrt{\frac{n_1\left(\sigma_1^2+d_1^2\right)+n_2\left(\sigma_2^2+d_2^2\right)}{n_1+n_2}},$$

where

$$d_1 = \bar{X}_1 - \bar{X} = 20-18 = 2 \quad \Rightarrow \quad d_1^2 = 4$$

$$d_2 = \bar{X}_2 - \bar{X} = 15-18 = -3 \quad \Rightarrow \quad d_2^2 = 9$$

$\therefore$

$$\sigma = \sqrt{\frac{90(64+4)+60(36+9)}{90+60}} = \sqrt{\frac{90\times68+60\times45}{150}}$$

$$= \sqrt{\frac{6120+2700}{150}} = \sqrt{\frac{8820}{150}} = \sqrt{58.8} = 7.67 \text{ (app.)}$$

EXAMPLE 29. *Find the missing information from the following:*

	Group I	*Group II*	*Group III*	*Combined*
Number	50	?	90	200
Standard Deviation	6	7	?	7.746
Mean	113	?	115	116

[Delhi Univ. B.Com. (H) 1992]

SOLUTION. Let n_1, n_2, n_3 denote the number of observations in Group *I*, Group *II*, Group *III* respectively. Then we are given

$$n_1 + n_2 + n_3 = 200$$

But $n_1 = 50$ and $n_3 = 90$ $\therefore$ $50 + n_2 + 90 = 200 \Rightarrow n_2 = 60$

Let $\bar{X}_1$, $\bar{X}_2$, $\bar{X}_3$ denote the means of Group *I*, Group *II*, Group *III* respectively. Then the combined mean, $\bar{X}$, is given by

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2 + n_3\bar{X}_3}{n_1 + n_2 + n_3}$$

$$\Rightarrow \quad 116 = \frac{50 \times 113 + 60 \times \bar{X}_2 + 90 \times 115}{200}$$

$$\Rightarrow \quad 23200 = 5650 + 60\ \bar{X}_2 + 10350$$

$$\Rightarrow \quad 60\ \bar{X}_2 = 23200 - 5650 - 10350 = 7200$$

$$\Rightarrow \quad \bar{X}_2 = \frac{7200}{60} = 120$$

Let σ_1, σ_2, σ_3 denote the standard deviations of Group *I*, Group *II*, Group *III* respectively. Then the combined standard deviation, σ, is given by

$$\sigma = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2) + n_3(\sigma_3^2 + d_3^2)}{n_1 + n_2 + n_3}}$$

$$\Rightarrow \quad (n_1 + n_2 + n_3)\sigma^2 = n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2) + n_3(\sigma_3^2 + d_3^2)$$

where

$$d_1 = \bar{X}_1 - \bar{X} = 113 - 116 = -3$$

$$d_2 = \bar{X}_2 - \bar{X} = 120 - 116 = 4$$

$$d_3 = \bar{X}_3 - \bar{X} = 115 - 116 = -1$$

$$\therefore \quad 200\,(7.746)^2 = 50\,(36 + 9) + 60\,(49 + 16) + 90\,(\sigma_3^2 + 1)$$

$$\Rightarrow \quad 200\,(60.000516) = 2250 + 3900 + 90 + 90\,\sigma_3^2$$

$$\Rightarrow \quad 12000 = 6240 + 90\,\sigma_3^2$$

$$\Rightarrow \quad 90\,\sigma_3^2 = 12000 - 6240 = 5760$$

$$\Rightarrow \quad \sigma_3^2 = \frac{5760}{90} = 64$$

$$\Rightarrow \quad \sigma_3 = 8 \qquad \text{(rejecting – ve value)}$$

Thus the missing figures are : $n_2 = 60$, $\bar{X}_2 = 120$ and $\sigma_3 = 8$.

EXAMPLE 39. *Calculate standard deviation and coefficient of variation from the data given below:*

Mid-point :	5	15	25	35	45	55	65	75
Frequency :	5	8	7	12	28	20	10	10

[Delhi Univ. B.Com. 1980]

SOLUTION.

CALCULATION OF STANDARD DEVIATION AND COEFFICIENT OF VARIATION

X	$u = \frac{X-45}{10}$ $(A = 45, h = 10)$	f	fu	fu^2
5	−4	5	−20	80
15	−3	8	−24	72
25	−2	7	−14	28
35	−1	12	−12	12
45	0	28	0	0
55	+1	20	+20	20
65	+2	10	+20	40
75	+3	10	+30	90
		$N = 100$	$\sum fu = 0$	$\sum fu^2 = 342$

Computation of arithmetic mean :

$$\bar{X} = A + \frac{\sum fu}{N} \times h = 45 + \frac{0}{100} \times 10 = 45$$

Computation of standard deviation :

$$\sigma = \sqrt{\frac{\sum fu^2}{N} - \left(\frac{\sum fu}{N}\right)^2} \times h = \sqrt{\frac{342}{100} - \left(\frac{0}{100}\right)^2} \times 10$$

$$= 1.849 \times 10 = 18.49$$

Computation of coefficient of variation :

$$C.V. = \frac{\sigma}{\bar{X}} \times 100 = \frac{18.49}{45} \times 100 = 41.09\%.$$

EXAMPLE 40. *Calculate standard deviation and coefficient of variation from the following data :*

Class Interval :	5 – 10	10 – 15	15 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency :	11	9	3	8	4	12	17	31

SOLUTION. Since class intervals are unequal, we shall use *c* (the common factor) instead of *h* in the formula.

CALCULATION OF STANDARD DEVIATION AND COEFFICIENT OF VARIATION

Class Interval	*Mid-value* X	$u = \frac{X-25}{5}$ $(A = 25, c = 5)$	f	fu	fu^2
5 – 10	7.5	– 3.5	11	– 38.5	134.75
10 – 15	12.5	– 2.5	9	– 22.5	56.25
15 – 20	17.5	– 1.5	3	– 4.5	6.75
20 – 30	25.0	0	8	0	0
30 – 40	35.0	2.0	4	8.0	16.00
40 – 50	45.0	4.0	12	48.0	192.00
50 – 60	55.0	6.0	17	102.0	612.00
60 – 70	65.0	8.0	31	248.0	1984.00
			$N = \sum f = 95$	$\sum fu = 340.5$	$\sum fu^2 = 3001.75$

Computation of Arithmetic Mean :

$$\bar{X} = A + \frac{\sum fu}{N} \times c = 25 + \frac{340.5}{95} \times 5 = 25 + 17.92 = 42.92$$

Computation of Standard Deviation :

$$\sigma = \sqrt{\frac{\sum fu^2}{N} - \left(\frac{\sum fu}{N}\right)^2} \times c = \sqrt{\frac{3001.75}{95} - \left(\frac{340.5}{95}\right)^2} \times 5$$

$$= \sqrt{31.597 - 12.847} \times 5 = 4.33 \times 5 = 21.65$$

Computation of Coefficient of Variation :

$$C.V. = \frac{\sigma}{\bar{X}} \times 100 = \frac{21.65}{42.92} \times 100 = 50.44\%.$$

EXAMPLE 41. *A purchasing agent obtained samples of lamps from two suppliers. He had the samples tested in his own laboratory for the lengths of life with the following results:*

Length of life (in hours)	Samples from	
	Company A	Company B
700 – 900	10	3
900 – 1100	16	42
1100 – 1300	26	12
1300 – 1500	8	3

(i) Which company's lamps have greater average life?

(ii) Which company's lamps are more uniform? **[Delhi Univ. B.Com. 2006]**

SOLUTION. **CALCULATIONS FOR MEAN AND STANDARD DEVIATION**

Length of life (in hours)	Mid-value X	$u = \frac{X-1000}{200}$ ($A = 1000, h = 200$)	Company A			Company B		
			f	fu	fu^2	f	fu	fu^2
700 – 900	800	–1	10	–10	10	3	–3	3
900 –1100	1000	0	16	0	0	42	0	0
1100 –1300	1200	1	26	26	26	12	12	12
1300 –1500	1400	2	8	16	32	3	6	12
			60	32	68	60	15	27

Company A :

$$\bar{X} = A + \frac{\sum fu}{N} \times h$$

$$= 1000 + \frac{32}{60} \times 200$$

$$= 1000 + 106.67 = 1106.67$$

$$\sigma = \sqrt{\frac{\sum fu^2}{N} - \left(\frac{\sum fu}{N}\right)^2} \times h$$

$$= \sqrt{\frac{68}{60} - \left(\frac{32}{60}\right)^2} \times 200$$

$$= \sqrt{1.133 - 0.284} \times 200$$

$$= \sqrt{0.849} \times 200 = 0.921 \times 200$$

$$= 184.2$$

$$C.V. = \frac{\sigma}{\bar{X}} \times 100 = \frac{184.2}{1106.67}$$

$$= 16.64\ \%$$

Company B :

$$\bar{X} = A + \frac{\sum fu}{N} \times h$$

$$= 1000 + \frac{15}{60} \times 200$$

$$= 1000 + 50 = 1050$$

$$\sigma = \sqrt{\frac{\sum fu^2}{N} - \left(\frac{\sum fu}{N}\right)^2} \times h$$

$$= \sqrt{\frac{27}{60} - \left(\frac{15}{60}\right)^2} \times 200$$

$$= \sqrt{0.45 - 0.0625} \times 200$$

$$= \sqrt{0.3875} \times 200 = 0.622 \times 200$$

$$= 124.4$$

$$C.V. = \frac{\sigma}{\bar{X}} \times 100 = \frac{124.4}{1050} \times 100$$

$$= 11.85\ \%$$

(i) The lamps of company A have greater average life.

(ii) The lamps of company B are more uniform, because $C.V.$ of samples of bulbs from company B is less than that of company A.